Abstract Title Page

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Title: Using Robust Variance Estimation to Combine Multiple Regression Estimates with Meta-Analysis

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Abstract Body

Limit 4 pages single-spaced.

Background / Context:

Description of prior research and its intellectual context.

Methods for meta-analysis have made great strides over the past 30 years. The focus of this work has been primarily on synthesizing univariate and bivariate effect size estimates. Increasingly, however, multivariate models such as multivariate linear models, factor analyses, and latent trait models are used to describe relationships in the social sciences and in medicine. Meta-analytic methods to combine effect sizes from these kinds of models are often cumbersome to implement or nonexistent.

Multiple regression is one of the most commonly used statistical methods in the social sciences. Because of the multivariate nature of multiple regression coefficients, they have historically been very difficult to combine with meta-analysis. Becker and Wu (2007) outline three key difficulties in combining multiple regression slope estimates. First, all model outcomes must be measured on a common scale. Second, the slope estimate of interest (focal slope) is measured on a common scale across studies. Finally, each study estimates the partial relationship between the focal slope and the outcome using the model (i.e. includes an identical set of additional predictors). Maintaining these assumptions in any given synthesis will almost always be impossible.

Purpose / Objective / Research Question / Focus of Study:

Description of the focus of the research.

The purpose of this study was to explore the use of robust variance estimation for combining commonly specified multiple regression models and for combining sample-dependent focal slope estimates from diversely specified models. The proposed estimator obviates traditionally required information about the covariance structure of the dependent effect size estimates, making it a potentially flexible method for conducing meta-analyses of regression estimates.

A series of Monte Carlo simulations were conducted to explore the performance of the robust variance estimator under different meta-analytic conditions.

Setting:

Description of the research location.
(May not be applicable for Methods submissions)

Not applicable.

Population / Participants / Subjects:

Description of the participants in the study: who, how many, key features, or characteristics.
(May not be applicable for Methods submissions)

Not applicable.

Intervention / Program / Practice:

Description of the intervention, program, or practice, including details of administration and duration.

(May not be applicable for Methods submissions)

Not applicable.

Significance / Novelty of study:
Description of what is missing in previous work and the contribution the study makes.

Researchers across different fields are clearly eager to make use of multiple regression estimates in meta-analytic fashion. Some of the methods in use are intuitive, others are considerably more complex. Many of the methods, however, fail to address the fundamental problems of combining multiple regression estimates: collinearity among model predictors and diverse model specification. Other methods, such as generalized least squares and individual participant data meta-analytic techniques are promising but often the data needed to use those methods are unavailable. A flexible alternative is needed to meta-analyze multiple regression estimates.

Statistical, Measurement, or Econometric Model:
Description of the proposed new methods or novel applications of existing methods.

Because the covariance structure of correlated effect size estimates is almost never report (or even explored) in primary research, Hedges, Tipton, and Johnson (2010) discuss in detail the use of robust standard errors in meta-regression with dependent effect size estimates. Their approach has been used with standardized mean differences and effect size estimates for binary outcomes. It has not yet been applied to multiple regression studies.

This study uses the robust variance estimator to combine commonly specified models across a pool of study samples and to combine multiple, sample-dependent, focal slope estimates derived from diversely specified models across study samples. More details of the estimator are provided in Appendix B.

Usefulness / Applicability of Method:
Demonstration of the usefulness of the proposed methods using hypothetical or real data.

Two sets of simulations studies were conducted. First, a series of focal slope meta-analyses were conducted. Here, a regression model was specified for each study sample (m), a common slope estimate (e.g. x_1) was extracted and the inclusion of additional study covariates in that model was recorded. As such, the vector of slope estimates to be combined with meta-analysis was derived from diversely specified models. And, each study sample estimated multiple (k) focal slopes. Second, a series of model-based meta-analyses were conducted. Here, m study samples were generated and a common model was specified for each sample. Each study sample, then, contributed a vector of slope estimates to the meta-analyses. The following parameter values were used in the simulations: n, is the primary study sample size; m, the number of study samples per meta-analysis; k, the number of slope estimates per study sample; ρ, the correlation among slope estimates; and I^2, the proportion of slope estimate heterogeneity. For each approach, a total of 243 parameter combinations were used. For each combination 1000 meta-analyses were conducted.

Each meta-analysis was conducted using inverse-variance weight methods (e.g. Hedges & Olkin, 1985) under fixed-effects model assumptions. Robust standard errors were computed for each mean slope estimate for each meta-analysis using simple fixed-effects weights. Parameter recovery and percent bias calculated for each meta-analysis to better understand the performance of the robust variance estimator.

Research Design:
Description of the research design (e.g., qualitative case study, quasi-experimental design, secondary analysis, analytic essay, randomized field trial).
(May not be applicable for Methods submissions)

Not applicable.

Data Collection and Analysis:
Description of the methods for collecting and analyzing data.
(May not be applicable for Methods submissions)

Not applicable.

Findings / Results:
Description of the main findings with specific details.
(May not be applicable for Methods submissions)

Overall, the robust standard errors recovered the parameter values specified in the simulated datasets. In the focal slope approach, the mean estimates were biased, consistently underestimating the parameter value. However, after a bias adjustment the robust confidence intervals recovered the population value at or above the specified nominal (95 percent) value. In all conditions, the model-based approach demonstrated close to or above 95 percent parameter recovery. Efficiency was largely a function of the number of studies included in the meta-analysis, *m*.

Conclusions:
Description of conclusions, recommendations, and limitations based on findings.

The results of the simulations for this approach were promising. The robust confidence intervals for each set of parameter combinations were close to nominal probability content in nearly all of the specified conditions. And, when they departed from nominal coverage, it was in the conservative direction. This was most notable when the ratio of studies *m* to the number of parameter estimates *k* was small.

The robust variance estimator outlined by Hedges, Tipton, and Johnson (2010) provides a flexible method for combining multivariate results. The results of this study indicate that using robust variance estimation is a viable alternative to the methods outlined by Wu and Becker (2007).

Appendices

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Appendix A. References

References are to be in APA version 6 format.

Becker, B. J., & Wu, M.-J. (2007). The synthesis of regression slopes in meta-analysis. *Statistical Science, 22*(3), 414.

Hedges, L. V., & Olkin, I. (1985). *Statistical methods for meta-analysis*. Orlando, FL: Academic Press.

Hedges, L. V., Tipton, E., & Johnson, M. C. (2010). Robust variance estimation in meta-regression with dependent effect size estimates. *Research Synthesis Methods*, *1*(1), 39–65.

Ishak, K. J., Platt, R. W., Joseph, L., & Hanley, J. A. (2008). Impact of approximating or ignoring within-study covariances in multivariate meta-analyses. *Statistics in medicine*, *27*(5), 670–686.

Appendix B. Tables and Figures

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Robust variance estimation and meta-analysis. Because the covariance structure of correlated effect size estimates is almost never report (or even explored) in primary research, Hedges, Tipton, and Johnson (2010) discuss in detail the use of robust standard errors in meta-regression with dependent effect size estimates. Their approach has been used with standardized mean differences and effect size estimates for binary outcomes. Their method may be applied to the synthesis of multiple regression slope estimates.

The procedures to follow assume that dependencies occur through correlated error terms. As such, a correlated effects modeling approach is presented. Modeling multiple regression estimates takes the following linear model form

$$\mathbf{b} = \mathbf{X}\boldsymbol{\beta} + \mathbf{e} \, ,$$

where **b** is a vector of n (from $n = 1$ to k) clustered partial effects in m studies (clusters), **X** is a design matrix of ($k_j \times p$) meta-regression covariates (e.g. model specification indicators), $\boldsymbol{\beta}$ is a vector of unknown meta-regression coefficients, and **e** is a vector of residuals. In the context of multiple regression estimates, **b** is a vector of stacked model

slope (and intercept) estimates. In the case of combining commonly specified models **X** may be an identity matrix and

$$\mathbf{b} = \begin{bmatrix} b_{01} \\ b_{11} \\ \vdots \\ b_{1p} \\ \vdots \\ b_{k0} \\ \vdots \\ b_{kp} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & 0 \\ 0 & 1 & \dots & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_p \end{bmatrix} + \begin{bmatrix} e_0 \\ e_1 \\ \vdots \\ e_p \end{bmatrix}.$$

The weighted least squares estimate of $\boldsymbol{\beta}$ is

$$\widehat{\boldsymbol{\beta}} = \left(\sum_{j=1}^{m} \mathbf{X}_j' \mathbf{W}_j \mathbf{X}_j\right)^{-1} \left(\sum_{j=1}^{m} \mathbf{X}_j' \mathbf{W}_j \mathbf{b}_j\right),$$

with variance

$$V = \left(\sum_{j=1}^{m} \mathbf{X}_j' \mathbf{W}_j \mathbf{X}_j\right)^{-1} \left(\frac{1}{m}\sum_{j=1}^{m} \mathbf{X}_j' \mathbf{W}_j \Sigma_j \mathbf{W}_j \mathbf{X}_j\right) \left(\sum_{j=1}^{m} \mathbf{X}_j' \mathbf{W}_j \mathbf{X}_j\right)^{-1},$$

where **W** is a weight matrix for the *m*th study such that $\mathbf{W} = \mathrm{diag}(\mathbf{W}_1 \ldots \mathbf{W}_m)$ such that $\mathbf{W}_j = \mathrm{diag}(w_{j1},\ldots,w_{jkj})$, Σ is the *m*th study's Cov(**b**) such that $\Sigma = \mathrm{diag}(\Sigma_1, \dots, \Sigma_m)$. Hedges et al. show that every element of Σ_j does not need to be estimated but rather the average of linear combinations of Σ_j need be estimated, specifically

$$\frac{1}{m}\sum_{j=1}^{m} \mathbf{X}_j' \mathbf{W}_j \Sigma_j \mathbf{W}_j \mathbf{X}_j .$$

Because study report information about Σ_j is almost never provided, an empirical estimate of Σ_j is needed. Hedges, Tipton, and Johnson (2010) show that the cross product of within-study residuals may be used as a crude approximation of Σ_j so that the robust variance estimator is

$$\mathbf{V}^{\mathrm{R}} = \left(\sum_{j=1}^{m} \mathbf{X}_j' \mathbf{W}_j \mathbf{X}_j\right)^{-1} \left(\frac{1}{m}\sum_{j=1}^{m} \mathbf{X}_j' \mathbf{W}_j \mathbf{e}_j \mathbf{e}_j' \mathbf{W}_j \mathbf{X}_j\right) \left(\sum_{j=1}^{m} \mathbf{X}_j' \mathbf{W}_j \mathbf{X}_j\right)^{-1},$$

where $\mathbf{e}_j \mathbf{e}_j'$ is the *j*th study's matrix of cross products of within-study residuals. Specifically,

$$\mathbf{e}_j = \mathbf{b}_j - \mathbf{X}_j \boldsymbol{\beta} .$$

The asymptotic distribution of *m* as it approaches ∞ is

$$\sqrt{m}\,(\bar{\mathbf{V}})^{-\frac{1}{2}}(\mathbf{b}-\boldsymbol{\beta}) \rightarrow \; N_p(\mathbf{0}, \mathbf{I}_p)\,,$$

where $\bar{\mathbf{V}} = m\mathbf{V}^R$. As *m* rises, $\mathbf{b} \sim N(\boldsymbol{\beta}, \mathbf{V}^R)$. The estimated robust standard error of b_j is then

$$S_j^R = \sqrt{\frac{mv_{jj}^R}{m-p}}\,,$$

where v_{jj}^R is the *j*th diagonal element of $\mathbf{V}^R$ and $\sqrt{\frac{m}{m-p}}$ is a finite population correction. A robust significance test of β_j follows a *t* distribution, specifically

$$t_j^R = \frac{b}{S_j^R}\,,$$

with *m-p* degrees of freedom. And, a robust confidence interval may be obtained using

$$b_j - c_{\alpha/2}S_j^R \leq \; \beta_j \leq b_j + c_{\alpha/2}S_j^R\,,$$

where c_α is the 1-α point on the *t* distribution with *m-p* degrees of freedom.

Hedges, Tipton, and Johnson (2010) identify three important features of this estimator. First, and most importantly, the covariance structure of effect size estimates is not needed. Second, parameter estimates converge on the target parameter as the number of studies, not the number of cases within studies, rises. And, the authors show that accurate standard errors are produced with as few as 10 to 20 studies. Third, this estimator is unbiased for any set of weights.

The authors show the calculation of the mean effect size estimate is

$$b_1 = \frac{\sum_{j=1}^m w_{1j}T_{1j}}{\sum_{j=1}^m w_{1j}}\,,$$

and with identical weighting procedures used for within-study estimates, the robust variance estimate becomes

$$V^R = \frac{\left(\sum_{j=1}^m w_{1j}\bar{b}_j - b_1\right)^2}{\left(\sum_{j=1}^m w_j\right)^2}\,,$$

where $\bar{b}$ is the *j*th cluster's unweighted mean effect size estimate, b_1 is the weighted mean effect size estimate from, and w_j is the weight assigned to the *j*th cluster of effect size estimates;

assumed to be uniform (i.e. common across within-study effect size estimates). When uniform weights are used V^R is equal to (m-1)/m^2 times the value of the typical variance. And, the robust standard error estimate S^R is equal to 1/m times the typical variance of $\bar{b}_j$ when uniform weights are used.

Weighting within-study estimate and cluster (study) total weights warrants consideration. Because between-study variance is likely, in most meta-analyses, to be much greater than within-study variance, an optimal weighting procedure is used. As such assigning each within-study estimate an equal weight would mean

$$w_{ij} = \frac{1}{k_j \bar{v}_{j*}} = \frac{1}{\sum_{i=1}^{k} v_{ij}},$$

where $\bar{v}_{j*}$ is the average of the effect size estimate variances in study j. The authors note that little precision is lost with this approach so long as the covariance among within-study estimates is reasonably high. If the covariance structure of the estimates is known or estimable, it could be used to improve efficiency; however, the advantage of this approach is that it requires no information about the covariance structure to produce accurate standard errors.

It is also possible to estimate between-study variances with this approach. Using fixed effects weights described above, the weighted residual sum of squares homogeneity statistic is

$$Q_E = \sum_{j=1}^{m} \mathbf{b}_j' \mathbf{W}_j \mathbf{b}_j - \left(\sum_{j=1}^{m} \mathbf{b}_j' \mathbf{W}_j \mathbf{b}_j\right)\left(\sum_{j=1}^{m} \mathbf{X}_j' \mathbf{W}_j \mathbf{b}_j\right)^{-1}\left(\sum_{j=1}^{m} \mathbf{X}_j' \mathbf{W}_j \mathbf{b}_j\right),$$

and the estimated residual variance component is

$$\hat{\tau}^2 = \frac{Q_E - m + \mathrm{tr}\left[\mathbf{V}\left(\sum_{j=1}^{m} \frac{w_j}{k_j} \mathbf{X}_j' \mathbf{X}_j\right)\right] + \rho * \mathrm{tr}\left[\mathbf{V}\left(\sum_{j=1}^{m} \frac{w_j}{k_j} [\mathbf{X}_j' \mathbf{J}_j \mathbf{X}_j - \mathbf{X}_j' \mathbf{X}_j]\right)\right]}{\sum_{j=1}^{m} k_j w_j - \mathrm{tr}\left[\mathbf{V}\left(\sum_{j=1}^{m} \mathrm{w}_j^2 \mathbf{X}_j' \mathbf{J}_j \mathbf{X}_j\right)\right]},$$

where $\mathbf{J}_j$ is a $k_j \times k_j$ matrix of 1's, ρ is the within study correlation of effects, and $\mathbf{V}$ is the inverse of the $\mathbf{X}'\mathbf{W}\mathbf{X}$ matrix specified as

$$\mathbf{V} = \left(\sum_{j=1}^{m} w_j \mathbf{X}_j' \mathbf{X}_j\right)^{-1}.$$

Knowing ρ may seem contradictory to this general approach, since we presume the covariance structure of dependent estimates is unknown. However, for this purpose, estimating a between study variance component, ρ may be roughly approximated without severe penalty so long as m is reasonably large. Ishak, Platt, Joseph, & Hanley (2008) simulated meta-regression analyses

under various scenarios that demonstrate this point, and Hedges, Tipton, and Johnson (2010) reinforce it in their work.